

$$1. \text{ a) } \frac{1}{3} + \frac{1}{2 \times 3} = \frac{1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}; \quad \frac{1}{4} + \frac{1}{3 \times 4} = \frac{1}{4} + \frac{1}{12} = \frac{3}{12} + \frac{1}{12} = \frac{4}{12} = \frac{1}{3}$$

$$\frac{1}{5} + \frac{1}{4 \times 5} = \frac{1}{5} + \frac{1}{20} = \frac{4}{20} + \frac{1}{20} = \frac{5}{20} = \frac{1}{4}$$

b) Pour n un nombre entier positif non nul, on a :

$$\frac{1}{n+1} + \frac{1}{n \times (n+1)} = \frac{n}{n \times (n+1)} + \frac{1}{n \times (n+1)} = \frac{n+1}{n \times (n+1)} = \frac{1 \times (n+1)}{n \times (n+1)} = \frac{1}{n}$$

c)

$$\text{Pour } n = 5, \text{ on a : } \frac{1}{5} = \frac{1}{5+1} + \frac{1}{5 \times (5+1)} = \frac{1}{6} + \frac{1}{30}; \quad \text{pour } n = 6, \text{ on a : } \frac{1}{6} = \frac{1}{6+1} + \frac{1}{6 \times (6+1)} = \frac{1}{7} + \frac{1}{42}$$

$$2. \text{ a) } \frac{1}{2} + \frac{1}{6} = \frac{3}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$$

$$\text{b) On a : } \frac{2}{3} = \frac{1}{2} + \frac{1}{6} \text{ et d'après 1.b. } \frac{1}{2} = \frac{1}{2+1} + \frac{1}{2 \times (2+1)} = \frac{1}{3} + \frac{1}{6} \text{ et d'après 1.c. } \frac{1}{6} = \frac{1}{7} + \frac{1}{42}$$

$$\text{D'où : } \frac{2}{3} = \frac{1}{3} + \frac{1}{6} + \frac{1}{7} + \frac{1}{42}$$

c) Pour p un nombre entier positif non nul, on a :

$$\frac{1}{2 \times p} + \frac{1}{6 \times p} = \frac{3}{3 \times 2 \times p} + \frac{1}{6 \times p} = \frac{3}{6 \times p} + \frac{1}{6 \times p} = \frac{3+1}{6 \times p} = \frac{4}{6 \times p} = \frac{2 \times 2}{2 \times 3 \times p} = \frac{2}{3 \times p}$$

d)

$$\text{Pour } p = 3, \text{ on a : } \frac{2}{9} = \frac{2}{3 \times 3} = \frac{1}{2 \times 3} + \frac{1}{6 \times 3} = \frac{1}{6} + \frac{1}{18}; \quad \text{pour } p = 5, \text{ on a : } \frac{2}{15} = \frac{2}{3 \times 5} = \frac{1}{2 \times 5} + \frac{1}{6 \times 5} = \frac{1}{10} + \frac{1}{30}$$

$$3. \text{ a) } \frac{7}{11} = \frac{14}{22} = \frac{11+2+1}{22} = \frac{11}{22} + \frac{2}{22} + \frac{1}{22} = \frac{1}{2} + \frac{1}{11} + \frac{1}{22}$$

$$\text{b) } \frac{5}{7} = \frac{10}{14} = \frac{7+2+1}{14} = \frac{7}{14} + \frac{2}{14} + \frac{1}{14} = \frac{1}{2} + \frac{1}{7} + \frac{1}{14}$$